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Damping Wiggler Section for CLIC

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Summary

Considering a damping ring where radiation damping, quantum excitation and intrabeam scattering only occur in a few long wiggler sections — the rest of the ring being transparent —, I show that by a proper choice of wiggler strength, wiggler period, beta function, and beam energy, the CLIC design emittances and damping rates may be attained.

1 Introduction

Recently, J. Jowett and H. Owen [1] have demonstrated that in conventional damping ring designs based on the TME lattice and on established optimization algorithms [2], the intrabeam scattering growth rates are much larger than the radiation damping rates. In addition, growth times for fast beam-ion instability and single-bunch electron-cloud instability are worryingly short (of the order of 1 μs, corresponding to a few turns, if not fractions of a turn) [1, 3].

In order to reduce the harmful effect of intrabeam scattering, one needs to raise the beam energy, which simultaneously implies a (much larger) increase in the ring circumference. If the effective damping time is held constant, and contributions from wigglers are neglected, the ring circumference increases as the third power of the beam energy.

J. Jowett has further shown that in a ring with LEP circumference and operated at 4 GeV energy, the intrabeam scattering growth rates are much reduced and typical IBS growth times are larger than 1000 s. The radiation damping times are even longer still.

The radiation damping in such a ring would only come from the wiggler sections. The same is true for intrabeam scattering and quantum excitation. Thus, effectively the arcs contribute neither to excitation nor to damping. In this note, we consider the obvious next question whether the CLIC design emittance could be achieved in a pure wiggler channel (with intermittent re-acceleration). The conditions which we derive represent a minimum requirement which must be fufilled by the wiggler section of any future damping ring design based on damping wigglers.

Table 1 lists a few damping-ring parameters. A large bunch length will help to reduce the effect of intrabeam scattering. Therefore, in Table 1 we quoted the maximum value, for which downstream bunch compression may still be possible [5].

2 Formulae for Growth Rates and Emittance

We assume that the vertical emittance is equal to a certain fraction of the horizontal one (due to betatron coupling, and residual vertical dispersion), namely $\epsilon_y = \kappa \epsilon_x$, with $\kappa = 0.007$ for the CLIC parameters of Table 1.

In the following we will focus on the horizontal emittance. Radiation damping in the wiggler can be expresssed as

$$\left.\frac{d\epsilon_x}{dt}\right|_{\rm SR} = -2\epsilon_x C_d J_x E^3 \left\langle \frac{1}{\rho^2} \right\rangle \tag{1}$$

where the square brackets denote an average over one wiggler period, ρ the bending radius, J_x (≈ 1) the damping partition number, and

$$C_d = \frac{c r_e}{3(m_e c^2)^3} \approx 2.1 \times 10^3 \ {\rm m^2 GeV^{-3}\ s^{-1}}. \tag{2}$$

Emittance growth due to quantum excitation is written as

$$\left.\frac{d\epsilon_x}{dt}\right|_{\rm QE} = c C_Q E^5 \left\langle \frac{\mathcal{H}_x}{\rho^3} \right\rangle \tag{3}$$

with $\mathcal{H}_x = \frac{1}{\beta_x}(D_x^2 + (\alpha_x D_x + \beta_x D_x')^2)$ the dispersion invariant, and

$$C_Q = \frac{55}{48\sqrt{3}} \frac{r_e \hbar c}{(m_e c^2)^6} \approx 2 \times 10^{-4} \ {\rm m^2 GeV^{-5}}. \tag{4}$$

T. Raubenheimer has derived the following approximate expressions for intrabeam scattering growth rates [1, 4]:

$$\frac{1}{\tau_\delta} = \frac{c r_e^2 N_b}{64 \gamma^3 \sigma_z \sigma_\delta^2} \frac{1}{\epsilon_x \epsilon_y} \left(\frac{\epsilon_x \epsilon_y}{\beta_x \beta_y} \right)^{1/4} \log \left(\frac{\sigma_y \gamma^2 \epsilon_x}{r_e \beta_x} \right) \tag{5}$$

$$\frac{1}{\tau_x} = \frac{\sigma_\delta^2}{\epsilon_x} \mathcal{H}_x \frac{1}{\tau_\delta} \tag{6}$$

$$\frac{1}{\tau_y} = \frac{\sigma_\delta^2}{\epsilon_y} \left(\mathcal{H}_y + \frac{\beta_y}{\gamma^2} \right) \frac{1}{\tau_\delta} \tag{7}$$

Table 1: Relevant damping ring parameters

variable	symbol	value
norm. horiz. emittance [mm mrad]	$\epsilon_{x,N}$	0.45
norm. vert. emittance [mm mrad]	$\epsilon_{y,N}$	0.003
rms bunch length [cm]	σ_z	1
bunch population	N_b	4.2×10^9
repetition rate [Hz]	$f_{\rm rep}$	100

Combining Eqs. (5) and (6) we can write the intrabeam scattering growth rate as

$$\left.\frac{d\epsilon_x}{dt}\right|_{\rm IBS} \approx \frac{c r_e^2 N_b}{64\gamma^3 \sigma_z \epsilon_x \epsilon_y} \left(\epsilon_x \epsilon_y\right)^{1/4} \; \frac{\mathcal{H}_x}{\beta_x^{1/4}\beta_y^{1/4}} \; \log\left(\frac{\sigma_y \gamma^2 \epsilon_x}{r_e \beta_x}\right) \tag{8}$$

Now the equilibrium emittance in the wiggler follows from the equation

$$\left.\frac{d\epsilon_x}{dt}\right|_{\rm SR} + \left.\frac{d\epsilon_x}{dt}\right|_{\rm QE} + \left.\frac{d\epsilon_x}{dt}\right|_{\rm IBS} = 0. \tag{9}$$

Inserting the above expression we obtain

$$2\epsilon_x C_d J_x E^3 \left\langle \frac{1}{\rho^2} \right\rangle = c C_Q E^5 \left\langle \frac{\mathcal{H}_x}{\rho^3} \right\rangle + \frac{c r_e^2 N_b}{64\gamma^3 \sigma_z \kappa^{3/4} \epsilon_x^{3/2}} \left\langle \frac{\mathcal{H}_x}{\left(\beta_x \beta_y\right)^{1/4}} \right\rangle \log\left(\frac{\sigma_y \gamma^2 \epsilon_x}{r_e \beta_x}\right). \tag{10}$$

Assuming $\beta_x \approx \beta_y$ and $\epsilon_y = \kappa \epsilon_x$, the logarithm in the formula for intrabeam scattering can be rewritten as

$$\log\left(\frac{\sigma_y \gamma^2 \epsilon_x}{r_e \beta_x}\right) \approx \log\left(\frac{\kappa^{1/2} \epsilon_{x,N}^{3/2}}{r_e}\right) + \log\left(\frac{\gamma^{1/2}}{\beta_x^{1/2}}\right). \tag{11}$$

The first term only depends on the design parameters, and amounts to 9.1. The second term evaluates to 4.5, assuming $E = 4$ GeV and β_x =1 m, and to 4.1 for $E = 2$ GeV and β_x =1 m. We will treat this logarithm as a constant, called 'Log', with a numerical value of about 13.

We next need to evaluate the expressions in square brackets by integrating over the wiggler. We consider a sinusoidal wiggler field

$$B(z) = B_w \cos k_p z \tag{12}$$

with dispersion

$$D(z) = \frac{1}{k_p^2 \rho_w}(1 - \cos k_p z), \tag{13}$$

where ρ_w denotes the minimum bending radius, $k_p = 2\pi/\lambda_p$ and λ_p the wiggler period. Assuming $\lambda_p \ll \beta_x$, and $\beta_x \approx \beta_y$, and introducing the bend angle per wiggler pole $\theta_w = 1/(\rho_w k_p)$, one finds

$$\int_w \frac{\mathcal{H}_x}{\rho^3} dz \quad \approx \quad N_w \frac{8}{15} \frac{\beta_x}{\rho_w^2} \theta_w^3 \tag{14}$$

$$\int_w \frac{1}{\rho^2} dz \quad \approx \quad \pi N_w \theta_w / \rho_w \tag{15}$$

$$\int_w \mathcal{H}_x \; dz \quad \approx \quad \pi N_w \beta_x \rho_w \theta_w \tag{16}$$

where N_w is the number of wiggler periods, and β_x, the beta function, is assumed to be constant across the wiggler.

We can now substitute these formulae in Eq. (10), and obtain

$$\epsilon_{x,N} = \frac{1}{\tilde{A}}\left[\tilde{B}\gamma^3\theta_w^2\frac{\beta_x}{\rho_w} + \tilde{C}\beta_x^{1/2}\rho_w^2\frac{1}{\gamma^{9/2}\epsilon_{x,N}^{3/2}}\right] \tag{17}$$

where we have introduced the three coefficients:

$$\begin{aligned} \tilde{A} &= 2\pi C_d J_x (m_e c^2)^3 \approx 1.8\times 10^{-6}\ \mathrm{m^2 s^{-1}} && (18)\\ \tilde{B} &= c C_Q (m_e c^2)^5 \frac{8}{15} \approx 1.1\times 10^{-19}\ \mathrm{m^3 s^{-1}} && (19)\\ \tilde{C} &= \frac{c r_e^2 N_b \pi}{64\sigma_z \kappa^{3/4}}\ \mathrm{Log} \approx 2.5\times 10^{-8}\ \mathrm{m^2\ s^{-1}}. && (20) \end{aligned}$$

Equation (17) shows that in order to obtain a small emittance, one would like to make β_x and θ_w as small as possible. The dependence on the other parameters is less clear.

We must also assure that the radiation damping time is reasonable. We rewrite Eq. (1) as

$$\frac{1}{\tau_x} \equiv \frac{1}{2}\frac{1}{\epsilon_x}\frac{d\epsilon_x}{dt}\bigg|_{\mathrm{SR}} = C_d J_x \gamma (m_e c^2)^3 \frac{B_w^2}{((B\rho)/\gamma)^2} \tag{21}$$

Denoting the repetition rate by f_{rep}, the train length (including kicker rise time) by l_{train}, and assuming that each train is stored for at least n_d damping times and that the wiggler section has a total fixed length l_{wiggler}, the minimum beam energy required follows from

$$\frac{n_d f_{\mathrm{rep}} l_{\mathrm{train}}}{\tau_x l_{\mathrm{wiggler}}} \geq 1 \tag{22}$$

or, assuming $J_x \approx 1$,

$$\gamma \geq \frac{n_d f_{\mathrm{rep}} (B\rho/\gamma)^2 l_{\mathrm{train}}}{C_d l_{\mathrm{wiggler}} (m_e c^2)^3 B_w^2}. \tag{23}$$

Note that this condition is independent of the ring circumference.

3 Two Scenarios

Starting from Eq. (17) we now consider two possible scenarios:

case 1: quantum excitation and intrabeam scattering contribute equally to the equilibrium emittance; and

case 2: the effect of intrabeam scattering is kept small compared with the quantum excitation.

3.1 Case 1: equal contribution from quantum excitation and IBS

In view of Eq. (17) we demand

$$\frac{\beta_x^{1/2}\gamma^{15/2}}{\rho_w^3}\ \theta_w^2 = \frac{\tilde{C}}{\tilde{B}\epsilon_{x,N}^{3/2}}, \tag{24}$$

and then solve for the equilbrium emittance, using again Eq. (17):

$$\epsilon_{x,N} = \left[\frac{2\ \tilde{C}}{\tilde{A}} \frac{\beta_x^{1/2} \rho_w^2}{\gamma^{9/2}} \right]^{2/5} . \tag{25}$$

Inserting the values for the emittance and for $\tilde{A}$ and $\tilde{C}$, we find

$$\frac{\beta_x^{1/2} \rho_w^2}{\gamma^{9/2}} = 4.9 \times 10^{-15}\ \mathrm{m}^{5/2}, \tag{26}$$

which can be rewritten as

$$\frac{\beta_x^{1/2}}{\gamma^{5/2} B_w^2} = 1.7 \times 10^{-9}\ \mathrm{m}^{1/2}\ \mathrm{T}^{-2}, \tag{27}$$

where B_w is the peak magnetic field.

Figure 1 displays the required wiggler peak field strength as a function of the beam energy for two different values of β_x.

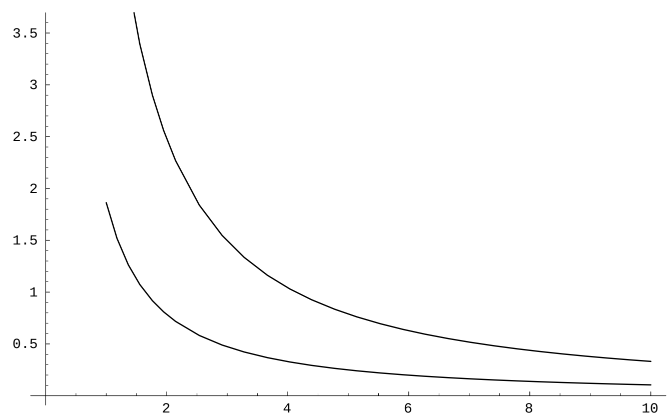

Figure 1: Required wiggler peak magnetic field in units of T as a function of beam energy in GeV, for $\beta_x = 100$ m (top curve) and $\beta_x = 1$ m (lower curve).

For a beam energy $E = 3$ GeV and $\beta_x = 30$ m, the wiggler field should be $B_w = 1.1$ T. Equation (24) then determines the wiggler period λ_p:

$$\lambda_p = \left(\frac{\tilde{C}(2\pi)^2 \rho_w^5}{\tilde{B} \epsilon_{x,N}^{3/2} \beta_x^{1/2} \gamma^{15/2}} \right)^{1/2} . \tag{28}$$

This gives a quite reasonable value of $\lambda_p = 0.13$ m.

Let us also estimate the damping time. Considering a train length $l_{\mathrm{train}} = 40$ m, a wiggler length $l_{\mathrm{wiggler}} = 500$ m, and $f_{\mathrm{rep}} = 100$ Hz, in order to store each train for at least $n_d = 5$ damping times, condition Eq. (23) requires a minimum γ of 120, *i.e.*, it is fulfilled. The damping rate in the wiggler is about 750 s^{-1}. It depends linearly on beam energy.

3.2 Case 2: IBS effect small compared with quantum excitation

This is perhaps the preferred situation. The two conditions describing this case follow from Eq. (17):

$$\beta_x^{1/2}\gamma^{5/2}\lambda_p^2 B_w^5 \gg (2\pi)^2\ (B\rho/\gamma)^5 \frac{\tilde{C}}{\tilde{B}\epsilon_{x,N}^{3/2}} \approx 4.4\times 10^8\ \mathrm{T}^5\ \mathrm{m}^{5/2} \tag{29}$$

and

$$B_w^3\lambda_p^2\beta_x = \epsilon_{x,N}\frac{\tilde{A}}{\tilde{B}}\ (2\pi)^2 \left(\frac{B\rho}{\gamma}\right)^3 \approx 1.5\ \mathrm{T}^3\mathrm{m}^3. \tag{30}$$

where $B\rho$ is the magnetic rigidity. Remarkably, Eq. (30) is independent of the beam energy. We choose $\beta_x = 30$ m and $\lambda_p = 0.15$ m, which requires $B_w = 1.3$ T.

Inserting these values into Eq. (29), we find that $\gamma \gg 10^4$, or energies larger than 5 GeV are necessary. At 6 GeV, the damping rate (inside the wiggler), Eq. (1), evaluates to 2000 s^{-1}, and condition (23) is easily fulfilled.

Could we make the damping even stronger, still retaining the correct equilibrium emittance and keeping intrabeam scattering small? A wiggler field of 5 T with a period of 2 cm would give the right equilibrium emittance. For a beam energy of 20 GeV, the radiation damping inside the wiggler would be $5\times 10^5\ \mathrm{s}^{-1}$, which appears sufficient to damp both ion and electron-cloud instabilities.

4 Fast Beam-Ion and Electron-Cloud Instabilities

For $\beta_x \approx \beta_y$ about constant, the growth rate of the fast beam-ion instability [7, 8, 9] can approximately be expressed as [10]

$$\frac{1}{\tau_{\mathrm{fbii}}} = \frac{p\sigma_{\mathrm{ion}}}{k_B T}\frac{N_b n_b r_e c}{\sqrt{18}\sqrt{\epsilon_{x,N}\epsilon_{y,N}}a}\frac{1}{\sqrt{Q}} \tag{31}$$

where σ_{ion} is the ionization cross section (for carbon monoxide this is taken to be 2 Mbarn), p the pressure, k_B Boltzmann's constant, T the temperature, $Q = 1$ the charge of the ion in units of e, n_b the number of bunches in the train ($n_b = 154$ for CLIC), and a the relative modulation amplitude of the ion oscillation frequency across a cell of the lattice (typically $a \approx 0.1$).

For CLIC parameters, the growth rate evaluates to $1/\tau_{\mathrm{fbii}} \approx 2\times 10^5\ \mathrm{s}^{-1}$. It only weakly depends on the lattice (via the parameter a and the assuption of equal beta functions), and it is independent of beam energy.

Growth rates for the electron-cloud instability [11] are estimated as

$$\frac{1}{\tau_{\mathrm{ecloud}}} = \frac{2N_b r_e c\beta_y}{\gamma h_x h_y L_{\mathrm{sep}}} \tag{32}$$

where L_{sep} denotes the bunch spacing, h_x the chamber half width, and h_y the chamber half height. With $h_x \approx h_y \approx 1$ cm, and $L_{\mathrm{sep}} = 0.2$ m, the growth rate is $10^6\ \mathrm{s}^{-1}$ at 6 GeV with $\beta_y = 30$ m.

Equation (32) assumes that the electron density reaches a saturation level determined by space charge forces, but the electron-cloud build up might be suppressed by the wiggler magnets, and/or by a judicious choice of the chamber aperture. The growth rate, Eq. (32), does not depend on the emittance. It decreases for smaller beta functions and higher beam energy.

5 Conclusion

We have shown that the CLIC design emittance and reasonable damping rates can be attained in a pure wiggler channel (with intermittent reacceleration), taking into account the combined effect of radiation damping, quantum excitation and intrabeam scattering. Example parameters, corresponding to our case studies, are compiled in Table 2. As expected by John Jowett and others, the beam energy tends to be higher than in conventional damping ring designs. High beam energies will also keep the contribution from intrabeam scattering smaller than that from quantum excitation, which might be the preferred situation. In addition, at beam energies of 20 GeV or higher, radiation damping will suppress the ion and electron-cloud instabilities.

Our parameter estimates were based on an approximative description of intrabeam scattering, Eqs. (5)—(7), and on approximations to the radiation and scattering integrals, Eqs. (14)–(16). The applicability of these approximations needs to be verified and possibly adjusted in a concrete optics design.

Table 2: Example wiggler parameters which produce the correct emittance and adequate damping times. Note that only for the parameters in the last column, the radiation damping rate is faster than the instability growth rates.

variable	case 1	case 2a	case 2b
beam energy [GeV]	3	6	20
wiggler peak field B_w [T]	1.1	1.3	5
wiggler period λ_p [m]	0.13	0.15	0.02
wiggler length $l_{\rm wiggler}$ [m]	500	500	500
beta function at wiggler [m]	30	30	30
ratio of growth rates $\tau_{\rm IBS}/\tau_{QE}$	1	2	500
damping rate inside wiggler [s^{-1}]	750	2000	5×10^5
ecloud growth rate $1/\tau_{\rm ecloud}$ [s^{-1}]	2×10^6	10^6	3×10^5
fast beam-ion growth rate $1/\tau_{\rm fbii}$ [s^{-1}]	2×10^5	2×10^5	2×10^5

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