Intra beam scattering in CLIC damping rings

A closed formula for intra beam scattering exists but its applicability to SPS/LEP type lepton ring is not immediately obvious. Let me examine the case.

The formula is valid for beams which are round (question!) and above transition (OK!). The formula is based on the Piwinski theory. It has been verified that this theory agrees with the Bjorken-Mtingwa theory above transition which is the reason for the second condition.

The emittances that we are concerned with here are 600 and 10 *nm* normalised *x,y* emittances, hence rather far from being round.

The basic Piwinski formulae are :

$$\frac{1}{\tau_p} = A\left(1-d^2\right)F_1$$

$$\frac{1}{\tau_x} = A\left(F_2 + d^2 F_1\right)$$

$$\frac{1}{\tau_y} = AF_3$$

The functions F_i are the scattering functions. The other parameters are :

$$A = \frac{n r_0^2 E_0}{64\pi\gamma\varepsilon_x\varepsilon_y\varepsilon_s},$$

where particle rest energy, radius and beam normalised emittances (1σ values) can be recognised.

Functions F_i depend on following parameters :

$$d^2 = \frac{1}{1+\dfrac{\beta_x\varepsilon_x}{\gamma\left(D\delta_p\right)^2}}, \quad a = \frac{\beta d}{D\gamma}, \quad b = a\sqrt{\frac{\varepsilon_x}{\varepsilon_y}}. \quad C = \frac{d}{D\sqrt{2r_0}}\left(\frac{\varepsilon\beta}{\gamma}\right)^{3/4}.$$

Parameter β with/without index is the optical (average) function, D the dispersion.

In our case *a* and *b* are small, while *d* is never much smaller than unity. From the nature of the functions F_i it follows that $F_{2,3}$ are very small and negative, while F_1 is much larger and positive such that it dominates the result for the longitudinal and horizontal growth rates.

If *a* and *b* are small then (according to the curves in the original Piwinski paper) :

$$F(a,b,w) = F(b,a,w) \approx F\left(\sqrt{ab},\sqrt{ab},w\right).$$

This then leads to the conjecture that the round beam formula may be used when parameters *a* and *b* are small enough (say less than 0.1). Parameters *a* and *b* are replaced by their geometrical average.

Hence :

$$\frac{1}{\tau_p} = AF\left(1-d^2\right)$$

$$\frac{1}{\tau_x} = AF\left(d^2 - \frac{a^2}{2}\right)$$

$$\frac{1}{\tau_y} = -AF\frac{a^2}{2}$$

$$F = 8\pi^2\left(\ln C - \frac{\bar{\gamma}}{2} + 2\right)\left(\frac{1}{a} - 1\right)e^{\frac{-4}{\pi}a^{0.6}}$$ ($\bar{\gamma}$ is Euler's constant)

Following table summarises the situation for LEP and SPS type damping rings .

		SPS		LEP	
E	*GeV*	1	5	1	5
ε_x	*nm*	300(600)	600	8(600)	600
ε_y	*nm*	10	10	1(10)	10
n	10^9	4	4	4	4
γ		1957	9785	1957	9785
D	*m*	2	2	0.66	0.66
β	*m*	47.2	47.2	53	53
δp	10^{-3}	0.03	0.15	0.016	0.078
σ_s	*mm*	2.4	6	2.3	6
ε_s	μeVs	0.76	48	0.38	24
d		0.6(0.45)	0.98	0.57(0.08)	0.67
a	10^{-3}	16(15)	6.5	40(9)	15
C		3(4)	2.6	0.6(2.3)	5.8
A		18(9)	0.03	13500(18)	0.06
F	10^3	12(14)	30	2(20)	16
τ_E	*ms*	0.007(0.01)	40	0.00005(0.003)	2

τ_x	*ms*	0.013(0.04)	1.2	0.0001(0.4)	2.4
τ_y	*s*	-0.034(-0.07)	-54	-0.00005(-0.065)	-9

For the 1 GeV case the equilibrium horizontal emittance was used first and the 600 *nm* as option.